The Shapley Value: Its Use and Implications on Internet Economics

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Outline

• **Network economics research and problems**

• **Two problems**
  – Design an *efficient* and *fair* profit-sharing mechanism for *cooperative* ISPs
  – Encourage *selfish* ISPs to operate at global *efficient/optimal* points
Building blocks of the Internet: ISPs

• The Internet is operated by thousands of interconnected Internet Service Providers (ISPs).

• An ISP is an autonomous business entity.  
  – Provide Internet services.  
  – Common objective: to make profit.
"When I was young, I thought money was the most important thing.

Now that I am old, ..........

I know it is the most important thing."
Three types of ISPs

1. **Eyeball (local) ISPs:**
   - Provide Internet access to residential users.
   - E.g. Time Warner Cable, Comcast, SingNet, MobileOne, Hutchinson

2. **Content ISPs:**
   - Server content providers and upload information.
   - E.g. Cogent, Akamai (Content Distribution Networks)

3. **Transit ISPs:**
   - Provide global connectivity, transit services for other ISPs.
   - E.g. Tier 1 ISPs: Level3, Qwest, Global Crossing
A motivating example

- **Win-win under Client-Server:**
  - Content providers find customers
  - Users obtain the content
  - ISPs generate revenue

Volume-based charge: $5/Gb

Fixed-rate charge: $50/month
What happens with P2P traffic?

- **Win-lose** under P2P paradigm:
  - Content providers pay less
  - Customers get faster downloads
  - Content ISP obtains less revenue
  - Eyeball ISPs handle more traffic

- In reality: P2P is a nightmare for ISPs.
Engineering solutions and ... beyond

• Proposed engineering approaches:
  – ISPs: Drop P2P packets based on port number
     – Users: Dynamic port selection
  – ISPs: Deep packet inspection (DPI)
     – Users: Disguise by encryption
  – ISPs: Behavioral analysis

• A new/global angle: Network Economics
  – Goal: a win-win/fair solution for all.
  – Issues:
    • What is a win-win/fair solution? How do we find it?
    • How do we design algorithms/protocols to achieve it?
  – Interdisciplinary research
    • Networking, Economics, Theoretic Computer Science
    • Operations Research, Regulatory Policy
Two important issues of the Internet

1. Network Neutrality Debate: Content-based Service Differentiations?

Legal/regulatory policy for the Internet industry: Allow or Not?
Allow: ISPs might over-charge; Not: ISPs have no incentive to invest.

In Oct 2007, Comcast was found to delay and block BitTorrent packets.

In Aug 2008, FCC ruled that Comcast broke the law.

In Apr 6th 2010, Federal court ruled that FCC didn’t have the authority to censure Comcast for throttling P2P packets.
AT&T’s iProblem

The iPhone has been a profit bonanza. But an overloaded network has AT&T at war with its customers.
The iPhone has swamped its data network and sparked a consumer rebellion. What can Ma Bell do?

Kendall-Heidrick, whose Steve Jobs, the father of Apple, steered that company to success, and whose deal with AT&T to put the iPhone on its wireless network was in the works in 2007. It seemed like a sure thing, a no-brainer.

Here was the deal: AT&T would pay $4 billion to buy the National Bell System, a subsidiary of AT&T, and would then be the exclusive provider of AT&T's wireless service. The iPhone would be locked to the network, making it impossible to use it on any other carrier.

But then came the unexpected. In 2008, AT&T announced it would be the exclusive carrier for the iPhone. The deal was sealed, and Apple and AT&T became partners.

By Roban Farazad
Illustration by Michael Wolins
Two important issues of the Internet

1. Network Neutrality Debate: Content-based Service Differentiations?

- Yes
- No

Legal/regulatory policy for the Internet industry: Allow or Not?
Allow: ISPs might over-charge; Not: ISPs have no incentive to invest. Either extreme will suppress the development of the Internet.

2. Network Balkanization: Break-up of connected ISPs

Not a technical/operation problem, but an economic issue of ISPs. Threatens the global connectivity of the Internet.
Other important problems that can be looked at

- A new/global angle: Network Economics
  - Goal: a win-win/fair solution
  - Issues: how do we design algorithms/protocols to achieve it?
Problems we try to solve

• Problem 1: Find a *win-win/fair* profit-sharing solution for ISPs.
• Challenges
  – What’s the solution? ISPs don’t know, even with best intentions.
  – How do we find it? Complex ISPs structure, computationally expensive.
  – How do we implement it? Need to be implementable for ISPs.
How do we share profit? -- the baseline case

- One content and one eyeball ISP
- Profit $V = \text{total revenue} = \text{content-side} + \text{eyeball-side}$
- Win-win/fair profit sharing:

$$\varphi_{B_1} = \varphi_{C_1} = \frac{1}{2} V$$
How do we share profit? -- two symmetric eyeball ISPs

Win-win/fair properties:

- **Symmetry**: same profit for symmetric eyeball ISPs
  \[ \phi_{B_1} = \phi_{B_2} = \phi_B \]

- **Efficiency**: summation of individual ISP profits equals V
  \[ \phi_{C_1} + 2\phi_B = V \]

- **Fairness**: same mutual contribution for any pair of ISPs
  \[ \phi_{C_1} - \frac{1}{2}V = \phi_{B_1} - 0 \]

Unique solution (Shapley value)

\[ \phi_{C_1} = \frac{2}{3}V \quad \phi_B = \frac{1}{6}V \]
Axiomatic characterization of the Shapley value

What is the Shapley value? – A measure of one’s contribution to different coalitions that it participates.
Stability of the Shapley value

- Convex game:
  - \( V(SUT) \geq V(S) + V(T) \)
  - Whole is bigger than the sum of parts.

\[
\begin{align*}
V\{1\} &= a, \quad V\{2\} = b \\
V\{1, 2\} &= c > a + b.
\end{align*}
\]
Stability of the Shapley value

- Convex game:
  - $V(SUT) \geq V(S) + V(T)$
  - Whole is bigger than the sum of parts.

- Core: the set of efficient profit-share that no coalition can improve upon or block.

$V(\{1\}) = a$, $V(\{2\}) = b$
$V(\{1,2\}) = c > a + b$. 
Stability of the Shapley value

• Convex game:
  – $V(SUT) \geq V(S) + V(T)$
  – Whole is bigger than the sum of parts.

• Core: the set of efficient profit-share that no coalition can improve upon or block.

• Shapley [1971]
  – Core is a convex set.
  – The value is located at the center of gravity of the core.

$V(\{1\}) = a$, $V(\{2\}) = b$
$V(\{1,2\}) = c > a + b$. 
How to share profit? -- n symmetric eyeball ISPs

• **Theorem**: the Shapley profit sharing solution is

\[
\phi_B = \frac{1}{n(n+1)} V, \quad \phi_C = \frac{n}{n+1} V
\]
Results and implications of profit sharing

\[ \varphi_B = \frac{1}{n(n+1)} V, \quad \varphi_C = \frac{n}{n+1} V \]

- With more eyeball ISPs, the content ISP gets a larger profit share.
  - Multiple eyeball ISPs provide redundancy,
  - The single content ISP has leverage.

- Content’s profit with one less eyeball:
  \[ \varphi_C = \frac{n-1}{n} V \]

- The marginal profit loss of the content ISP:
  \[ \Delta \varphi_C = \frac{n-1}{n} V - \frac{n}{n+1} V = -\frac{1}{n^2} \varphi_C \]

If an eyeball ISP leaves
  - The content ISP will lose \( \frac{1}{n^2} \) of its profit.
  - If \( n=1 \), the content ISP will lose all its profit.
Profit share -- multiple eyeball and content ISPs

- **Theorem:** the Shapley profit sharing solution is

\[ \phi_B = \frac{m}{n(n+m)}V, \quad \phi_C = \frac{n}{m(n+m)}V \]
Results and implications of ISP profit sharing

Each ISP’s profit share is

- Inversely proportional to the number of ISPs of the same type.
- Proportional to the number of ISPs of the other type.

Intuition

- When more ISPs provide the same service, each of them obtains less bargaining power.
- When fewer ISPs provide the same service, each of them becomes more important.

\[
\phi_B = \frac{m}{n} \frac{V}{(n+m)}, \quad \phi_C = \frac{n}{m} \frac{V}{(n+m)}
\]
Profit share -- eyeball, transit and content ISPs

- **Theorem**: the Shapley profit sharing solution is

\[
\phi_B = \frac{V}{n+m+k} \sum_{\mu=1}^{m} \sum_{\kappa=1}^{k} \left( \binom{m}{\mu} \binom{k}{\kappa} \frac{1}{\binom{n+m+k-1}{\mu+\kappa}} \right)
\]

\[
\phi_C = \frac{V}{n+m+k} \sum_{\nu=1}^{n} \sum_{\kappa=1}^{k} \left( \binom{n}{\nu} \binom{k}{\kappa} \frac{1}{\binom{n+m+k-1}{\nu+\kappa}} \right)
\]

\[
\phi_T = \frac{V}{n+m+k} \sum_{\mu=1}^{m} \sum_{\nu=1}^{n} \left( \binom{m}{\mu} \binom{n}{\nu} \frac{1}{\binom{n+m+k-1}{\mu+\nu}} \right)
\]
Profit share – general topologies

1. Shapley values under sub-topologies:

\[ \varphi_{C1} = 0 \]

\[ \varphi_{C1} = \frac{1}{3}V \]

2. Whether the profit can still be generated:

C_1 is veto.

\[ \varphi_{C1} = \frac{1}{4} \left[ 0 + \frac{1}{3}V + \frac{1}{3}V + V \right] = \frac{5}{12}V \]
Current ISP Business Practices: A Macroscopic View

Two forms of bilateral settlements:

Zero-Dollar Peering

Provider ISPs

Customer-Provider Settlement

Customer ISPs
Achieving the win-win/fair profit share
Achieving the win-win/fair profit share

- Two revenue flows to achieve the Shapley profit share:
  - Content-side revenue: Content $\rightarrow$ Transit $\rightarrow$ Eyeball
  - Eyeball-side revenue: Eyeball $\rightarrow$ Transit $\rightarrow$ Content
Achieving the Shapley solution by bilateral settlements

- When $CR \approx BR$, bilateral implementations:
  - Customer-Provider settlements (Transit ISPs as providers)
  - Zero-dollar Peering settlements (between Transit ISPs)
  - Current settlements can achieve fair profit-share for ISPs.
• If CR >> BR, bilateral implementations:
  – **Reverse Customer-Provider** (Transits compensate Eyeballs)
  – **Paid Peering** (Content-side compensates eyeball-side)
  – New settlements are needed to achieve fair profit-share.
Recap: ISP Practices from a Macroscopic View

Two current forms of bilateral settlements:
Our Implication: Two new forms of bilateral settlements:

- Zero-Dollar Peering
- Paid Peering
- Reverse Customer-Provider Settlement
- Customer-Provider Settlement

Next, a Microscopic View to inspect on ISP behaviors
Problems we try to solve

• Problem 1: **Find a win-win/fair profit-sharing solution for ISPs.**
  • Challenges
    – What’s the solution? ISPs don’t know, even with best intentions.
    – **Answer:** The Shapley value.
    – How do we find it? Complex structure, computationally expensive.
    – **Result:** Closed-form solution and Dynamic Programming.
    – How do we implement it? Need to be implementable for ISPs.
    – **Implication:** Two new bilateral settlements.

• Problem 2: **Encourage ISPs to operate at an efficient/optimal point.**
  • Challenges
    – How do we induce good behavior? Selfish behavior may hurt others ISPs.
    – What is the impact on the entire network? Equilibrium is not efficient.
Current ISP Business Practices: A Micro Perspective

Three levels of ISP decisions

- **Bilateral settlements** $\phi$
- **Routing decisions** $R$ (via BGP)
- **Interconnecting decision** $E$

Settlement $\phi$ affects $E$, $R$

Provider ISP

Customer-Provider Settlements

Interconnection withdrawal

Route over-charges

Interconnection withdrawal

Zero-Dollar Peering

Shortest Path Routing
Our solution: The Shapley Mechanism

Recall: three levels of ISP decisions

- Bilateral settlements
- Routing decisions $R$
- Interconnecting decision $E$

collects revenue from customers

distributes profits to ISPs

Settlement affects $E, R$

Bilateral settlements

Routing decisions $R$

Interconnecting decision $E$

Provider ISP

Customer ISP

Source

Destination

Zero-Dollar Peering

$$\varphi(\mathbf{E}, \mathbf{R})$$

Customer-Provider Settlements

$$\text{Provider ISP} \to \text{Customer ISP}$$

$$\text{Customer ISP} \to \text{Source}$$

$$\text{Destination} \to \text{Customer ISP}$$

$$\text{Customer ISP} \to \text{Destination}$$

$$\text{Source} \to \text{Provider ISP}$$

Google

YouTube

Akamai
ISP behaviors under the Shapley mechanism \( \varphi \)

**Profit distribution mechanism:** \( \varphi \)

**Local decisions:** \( E_i, R_i \)

**Objective:** to maximize \( \varphi_i(E, R) \)
Results: Incentives for using Optimal Routes

- Given any fixed interconnecting topology $E$, ISPs can locally decide routing strategies $\{R_i^*\}$ to maximize their profits.

- **Theorem** (Incentive for routing): Any ISP $i$ can maximize its profit $\varphi_i$ by locally **minimizing** the **global** routing cost.
  - Implication: ISPs adapt to global min cost routes.

- **Corollary** (Nash Equilibrium): Any global min cost routing decision is a Nash equilibrium for the set of all ISPs.
  - Implication: global min cost routes are stable.

**Surprising!** Local selfish routing behavior coincides with the globally optimal solution!
Results: Incentive for Interconnecting

- For any topology, a global optimal route $R^*$ is used by all ISPs. ISPs can locally decide interconnecting strategies $\{E_i^*\}$ to maximize their profits.

- **Theorem** (Incentive for interconnecting): After interconnecting, ISPs will have non-decreasing profits.
  
  - Implication: ISPs have incentive to interconnect.
  - Does not mean: All pairs of ISPs should be connected.
    - Redundant links might not reduce routing costs.
    - Sunk cost is not considered.

Local selfish interconnecting decisions form a globally optimal topology!
Problems we try to solve

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  Challenges
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  – Implication: Two new bilateral settlements

• Problem 2: Encourage ISPs to operate at an \textit{efficient/optimal} point.

  Challenges
  – How do we induce good behavior? Selfish behavior hurts other ISPs.
  – Answer: Shapley profit-distribution mechanism
  – Result: Incentives for optimal routes and interconnection
  – What is the impact on the entire network? Equilibrium is not efficient.
  – Result: Globally optimal/efficient equilibrium
A Game-theoretic framework

- Non-cooperative Game Theory
- Mechanism Design: e.g. Tax Policy, Auction Theory
- Coalition Game Theory

- New cooperative applications
  - Data Centers Networks
  - QoS supported services
Questions?

Thank you very much!